DEMONSTRATION OF PROP. XXV. OF THE FIRST BOOK OF EUCLID.

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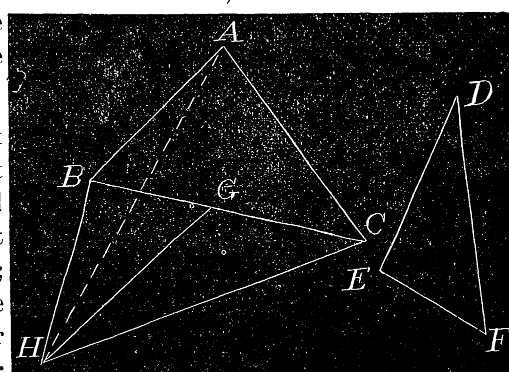
THE twenty-fifth proposition of the first book of Euclid, which that author (and, I believe, every succeeding geometer) demonstrates indirectly, that is by the *reductio ad absurdum*, may be demonstrated directly as follows:

Theorem:—If two triangles have two sides of the one equal to two sides of the other, each to each, but the bases unequal, the angle contained by the sides of that which has the greater base will be greater than the angle contained by the sides of the other.

In the triangles ABC, DEF let $AB = DE$, $AC = DF$ and BC be greater than EF, then shall the angle BAC be greater than the angle EDF.



Of the sides AB, AC let AB be the one which is not greater than the other, and from the base BC cut off a part $BG = EF$ and adjacent to AB; on BG construct the triangle BHG so that $BH = DE$ (or AB) and $HG = DF$ (or AC); join AH and HC. Because $BH = BA$, the angle BHA = the angle BAH. And since the side BH is not greater than HG the angle HGB is not greater than HBG; but the angle HGC is greater than HBG; therefore the angle HGC is greater than HGB; but HGB is greater than HCG; much more, then, is the angle HGC greater than HCG. And because the angle HGC is gaeater than HCG the side HC is greater than the side HG or its equal AC. Then, in the triangle HCA, the angle HAC is greater than the angle AHC because the side HC is greater than AC. To these unequal angles add the equals BHA and BAH and there results angle BAC greater than angle BHC; but $BHG = EDF$; therefore the angle BAC is greater than the angle EDF; which was to be proved.

QUERY, BY THE EDITOR. — As we have been requested to furnish a demonstration to the following propositon, the subjoined demonstration is submitted; and we present to our readers the query: By whom was the proposition originally announced?—Also, give the author's demonstration.